

Important Formulas;

Design of Machine Elements

UNIT-1

Normal stress on oblique plane;

$$\sigma_n = \left(\frac{\sigma_x + \sigma_y}{2} \right) - \left(\frac{\sigma_x - \sigma_y}{2} \right) \cos 2\theta - \tau_{xy} \sin 2\theta$$

Shear stress on oblique plane;

$$\tau = \left(\frac{\sigma_x - \sigma_y}{2} \right) \sin 2\theta - \tau_{xy} \cos 2\theta$$

Principle plane angle;

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

Maximum or Major Principle Stress;

$$\sigma_{max} = \left(\frac{\sigma_x + \sigma_y}{2} \right) + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2} \right)^2 + \tau_{xy}^2}$$

Minimum or Minor Principle stress;

$$\sigma_{min} = \left(\frac{\sigma_x + \sigma_y}{2} \right) - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2} \right)^2 + \tau_{xy}^2}$$

Maximum shear stress;

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2} \right)^2 + \tau_{xy}^2}$$

Factor of Safety;

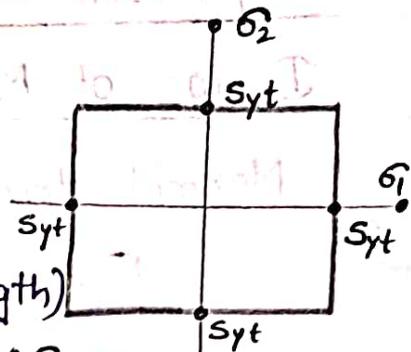
$$FOS = \frac{\text{Failure or Max}^m \text{ Stress}}{\text{Working Stress}}$$

Theory of Failure

1. Maximum Principle stress theory,

$$\sigma_{\max} \leq S_{yt} \quad (\text{Yield tensile strength})$$

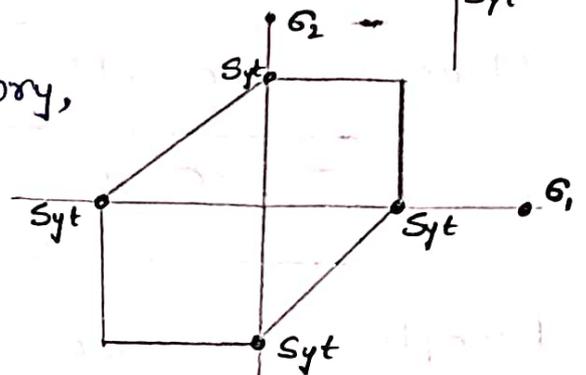
$$\sigma_{\max} \leq S_{ut} \quad (\text{Ultimate shear strength})$$



2. Maximum shear stress theory,

$$\tau_{\max} \leq \tau_{\text{permissible}}$$

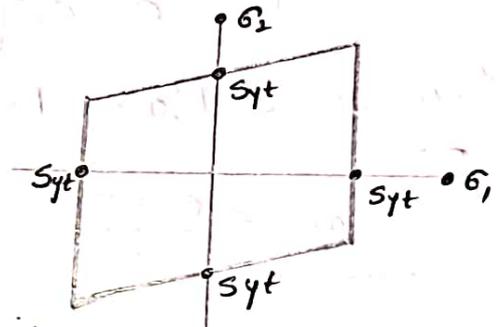
$$\sigma_1 - \sigma_2 \leq S_{yt}$$



3. Maximum principle strain theory (Saint-Venant Theory)

$$\epsilon_{\max} \leq \epsilon_{\text{permissible}}$$

$$\sigma_1 - \frac{\sigma_2}{m} \leq S_{yt}$$

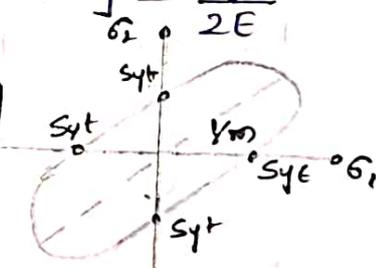


4. Maximum strain energy theory (Haigh theory)

$$U = \frac{1}{2E} \left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \frac{2(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)}{m} \right] \leq \frac{S_{yt}^2}{2E}$$

for biaxial,

$$\frac{1}{2E} \left[\sigma_1^2 + \sigma_2^2 - \frac{2\sigma_1\sigma_2}{m} \right] \leq \frac{S_{yt}^2}{2E}$$

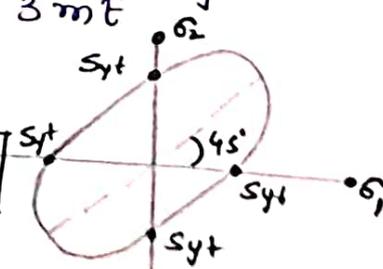


5. Maximum Distortion energy theory (Von-mises theory)

$$\frac{1+m}{6mE} \left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right] \leq \frac{1+m}{3mE} S_{yt}^2$$

for biaxial,

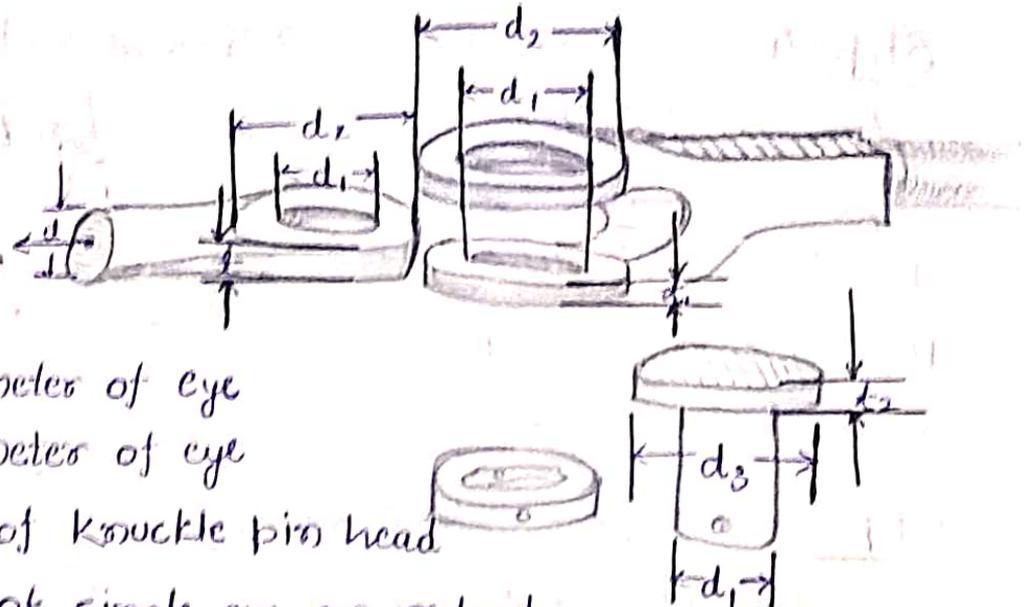
$$\sigma_1^2 + \sigma_2^2 - \sigma_1 \cdot \sigma_2 \leq S_{yt}^2$$



UNIT-2

Knuckle Joint :

d - dimension of circular rod (Diameter)



d_1 - Inner diameter of eye

d_2 - Outer diameter of eye

d_3 - Diameter of knuckle pin head

t - thickness of single eye or rod end

t_1 - thickness of double eye or fork end

t_2 - thickness of knuckle pin head

Step-1 Calculate the diameter of rod, by applying the relation,

$$\sigma_t = \frac{\text{Tensile load}}{\text{Resisting Area}}$$

$$\sigma_t = \frac{P}{\frac{\pi d^2}{4}}, \quad d = \sqrt{\frac{4P}{\pi \sigma_t}}$$



Step-2 Calculate the dimension of knuckle joint assembly parts by these empirical relation;

i). Inner diameter of eye, $d_1 = d$

ii). Outer diameter of eye, $d_2 = 2d$

iii). Diameter of knuckle pin head, $d_3 = 1.5d$

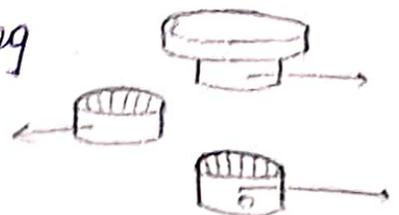
iv). Thickness of single eye, $t = 1.25d$

v). Thickness of double eye, $t_1 = 0.75d$

vi). Thickness of knuckle pin head, $t_2 = 0.5d$

Step-3 ~~Calculate the diameter~~ check the dimension of failure of knuckle pin due to shearing

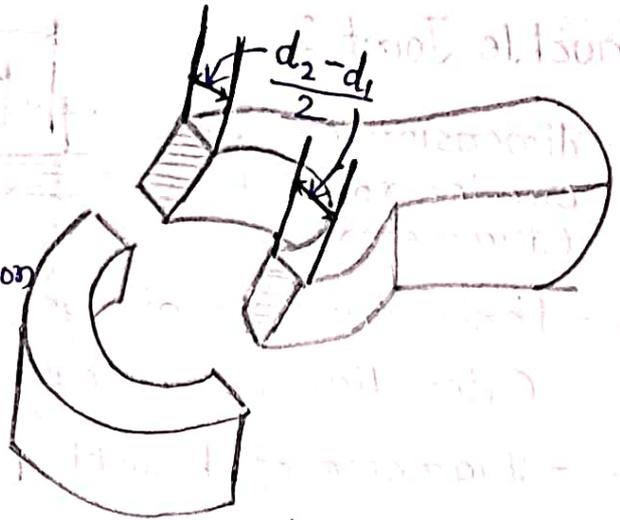
$$T_{cal} = \frac{2P}{\pi d_1}; \quad \text{If } T_{cal} \leq T_{given}, \text{ then dimension is safe.}$$



Step-4 Check the condition of failure in single eye due to tension.

$$\sigma_{t\text{cal}} = \frac{P}{t(d_2-d_1)}$$

If $\sigma_{t\text{cal}} \leq \sigma_t$ given, then dimension is in safe design.



Step-5 Check the condition of failure in single eye due to shearing.

$$\tau_{\text{cal}} = \frac{P}{t(d_2-d_1)}$$

If $\tau_{\text{cal}} \leq \tau$ given, then dimension is in safe design.

Step-6 Check the condition of failure in single eye due to crushing.

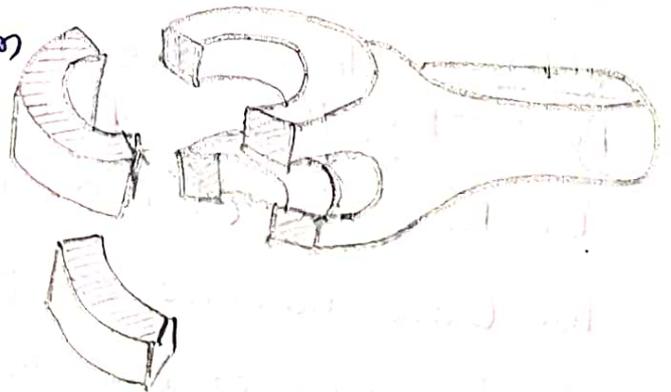
$$\sigma_{c\text{cal}} = \frac{P}{d_1 \times t}$$

If $\sigma_{c\text{cal}} \leq \sigma_c$ given, then dimension is in safe design.

Step-7 Check the condition of failure due to tension.

$$\sigma_{t\text{cal}} = \frac{P}{2(d_2-d_1)t}$$

If $\sigma_{t\text{cal}} \leq \sigma_t$ given, then dimension is in safe design.



Step-8 Check the condition of failure due to shearing.

$$\tau_{\text{cal}} = \frac{P}{2(d_2-d_1)t}$$

If $\tau_{\text{cal}} \leq \tau$ given, then dimension is in safe design.

Step-9 check the condition of failure due to crushing

$$\sigma_{cal} = \frac{P}{2t, d_1}$$

If $\sigma_{cal} \leq \sigma_{given}$, then dimension is in safe design.

Step 10 Calculate the total length of knuckle.

$$l = 4d$$

Lever:

Step 1 Calculate the diameter (d)

of rod or shaft by applying the relation.

$$T = P \times L$$

where $P \cong 800 \text{ N or } 400 \text{ N}$
 L - effective length of lever.

and from torsional equation,

$$T = \frac{\pi}{16} d^3 \tau$$

Step 2 Calculate the diameter of boss (d_2) and thickness (t_2) and length (l_2) applying relation;

$$d_2 = 1.6d$$

$$t_2 = 0.3d$$

$$l_2 = 1.25d$$

Step 3 Calculate the diameter of shaft at the centre of bearing (d_1)

Calculate the equivalent torque

$$T_e = \sqrt{M^2 + T^2}$$

$$T_e = \frac{\pi}{16} d_1^3 \times \tau_{given}$$

for bending moment
 $M = P \times l$, $l = 2l_2$
twisting moment
 $T = P \times L$ (given)

Step-4 Design of key ;

◦ Width of key, $w = \frac{d}{4}$

◦ thickness of key, $t = \frac{d}{6}$

◦ length of key, $l_1 = l_2$

or $T = \tau \omega l_1 \times \frac{d}{2}$ or $T = \sigma_c \times t \times l \times \frac{d}{4}$

Life of Antifriction Bearing ;

Life = Total number of revolution

or $L_{90/10} = \frac{60 N L_{10hr}}{10^6}$

average life, $L_{50} = 5L_{90}$

Load Life Relation ;

$L_{90} \text{ or } L_{10} = \left[\frac{C}{P} \right]^n$

Load carrying capacity ;

$C = P [L_{10}]^{1/n}$

where,

L_{90} or L_{10} - rated life in Mrev.

$L_{90} \text{ or } L_{10} = \frac{60 N L_{10hr}}{10^6}$

C - dynamic load carrying capacity

$n = k = p = 3$ (for ball bearing)
 $= 10/3$ (for roller bearing)

P = equivalent load

$P = V \times F_r + Y F_a$, F_r & F_a is given

F_r - Radial load, F_a - Axial load

V - Race rotation factor

= 1, inner race is rotating

= 1.2, outer race is rotating

UNIT-3

Design Procedure of shaft :

Step 1 Calculate the torque transmitted by shaft.

$$T = \frac{60P}{2\pi N}$$

P - Power

N - Speed in rpm

T - Torque transmitted

Step-2 Calculate the diameter from torsional equation,

$$T = \frac{\pi}{16} d^3 \tau$$

for solid shaft

$$T = \frac{\pi (D^4 - d^4) \tau}{16 D}$$

for hollow shaft

Step-3 Calculate the diameter of shaft on rigidity criteria,

$$T = \frac{G\theta}{l} \times \frac{\pi d^4}{32}$$

for solid shaft

$$T = \frac{G\theta}{l} \times \frac{\pi (D^4 - d^4)}{32}$$

for hollow shaft

Design Procedure of rectangular sunk key :

Step-1 Calculate torque transmitted by shaft

$$T = \frac{60P}{2\pi N}$$

Step 2 Calculate diameter of shaft from torsional eqⁿ

$$T = \frac{\pi d^3 \tau}{16}$$

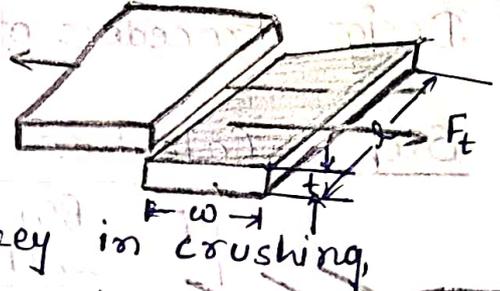
Step 3 Calculate the width and thickness of key

$$t = \frac{d}{6}$$

$$w = \frac{d}{4}$$

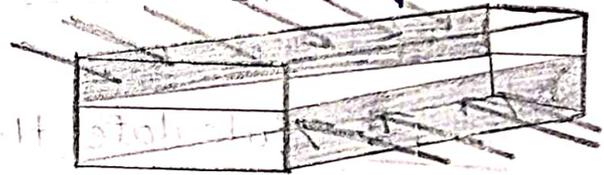
Step 4 Calculate the length of key in shearing,

$$T = T_{key} \times w \times l \times \frac{d}{2}$$



Step 5 Calculate the length of key in crushing,

$$T = \sigma_c \times t \times L \times \frac{d}{4}$$



Shaft strength factor (e), "Ratio of strength of shaft with keyway to strength of shaft without keyway."

$$e = 1 - 0.2 \left(\frac{h}{d} \right) - 1.01 \left(\frac{h}{d} \right)^2$$

Equivalent Torque,

$$T_e = \sqrt{M^2 + T^2}$$

Equivalent moment

$$M_e = \frac{1}{2} [M + \sqrt{M^2 + T^2}]$$

Design of Flange Coupling (Protected Type)

Let,

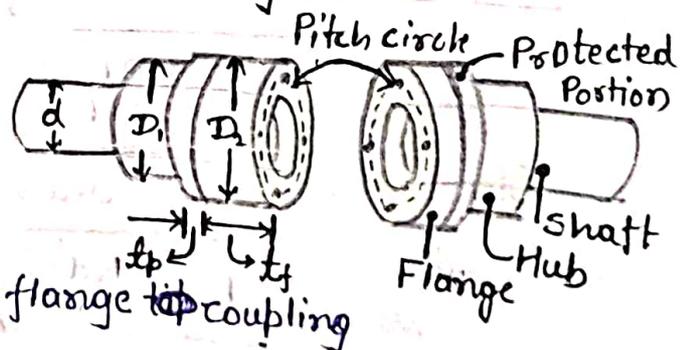
d = diameter of shaft

D_1 = Diameter of hub

D_2 = Outer diameter of flange of coupling

t_f = thickness of flange

t_p = thickness of protected portion.



A. Design of shaft and hub,

Step 1 Calculate torque transmitted by shaft,

$$P = \frac{2\pi NT}{60} \rightarrow ?$$

Step 2 Calculate the diameter of shaft,

i). from torsional strength,

$$T = \frac{\pi}{16} d^3 \tau$$

ii). from rigidity criteria,

$$T = \frac{G\theta}{l} \times \frac{\pi}{32} d^4$$

Step 3 Calculate the inner diameter of hub,

$$d_i = d$$

Step 4 Calculate the outside diameter of hub,

$$D = 2d$$

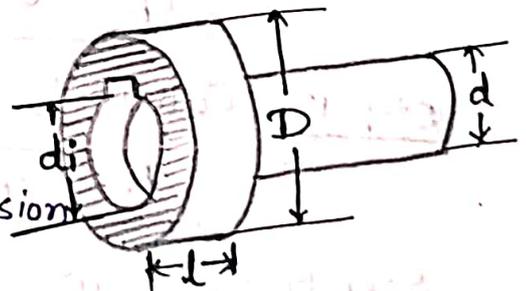
Step 5 Calculate the length of hub,

$$l = 1.5d$$

Step 6 Check the dimension of hub, assuming fail in shearing,

$$T = \frac{\pi}{16} \left(\frac{D^4 - d_i^4}{D} \right) \tau_{Hub}$$

If $T_{Hub} < T_{Hub}(\text{given})$ then dimension is in safe design.



B. Design of Key,

Step 7 Calculate width and thickness of key,

$$w = \frac{d}{4} \text{ \& } t = \frac{d}{6}$$

for rectangular key

$$w = t = \frac{d}{4}$$

for square key

Step-8 Calculate the length of key

$$l_k = l_H = 1.5 d$$

Step-9 Check shear stress induced in key,

$$T = \tau_{key} \times w \times l \times \frac{d}{2}$$

If $\tau_{key} < \tau_{key}(\text{given})$ then dimension is in safe design.

Step 10 Check the condition of failure in key due to crushing,

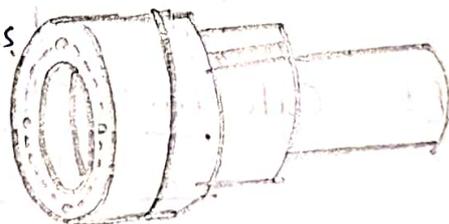
$$T = \sigma_c \times t \times l \times \frac{d}{2}$$

If $\sigma_c < \sigma_c(\text{given})$ then dimension is in safe design.

C. Design of flange portion;

Step 11 Calculate the thickness of flange (t_f)

$$t_f = 0.5 d$$



Step 12 Calculate the thickness of protected portion,

$$t_p = 0.25 d$$

Step 13 Calculate the outside diameter of flange,

$$D_2 = 4 d$$

Step 14 Calculate the pitch circle diameter,

$$D = 3 d$$

Step 15 Check the shear stress induced in flange,

$$T = \tau_{flange} \times \pi D \times t_f \times D$$

If $\tau_{flange} < \tau_{flange}(\text{given})$ then the dimension are in safe design.

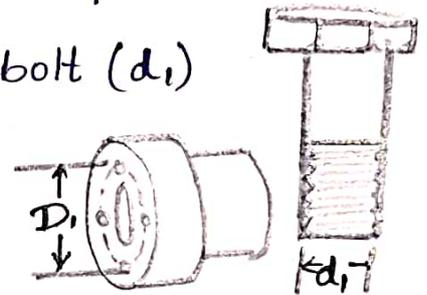
D. Design of nut and Bolt,

Step 16 Calculate the number of bolts according to international standard (I.S. 1990-1996)

Dia. of shaft	upto 40mm	upto 100 mm	upto 180 mm
no. of bolts	3	4	5

Step 17 Calculate the diameter of bolt (d_1)

$$T = \eta \times \frac{\pi^2}{d_1} \times \frac{D_1}{2}$$



Gear Terminology

o Module = $\frac{\text{Pitch circle diameter}}{\text{No. of Teeth}} = \frac{D}{T} = \frac{2R}{T}$

o Diametral Pitch = $\frac{\text{No. of Teeth}}{\text{Pitch circle dia}} = \frac{T}{D}$; $P_d = \frac{1}{m}$

o Circular Pitch, $P_c = \frac{\pi D}{T} = \pi m$; $P_c \times P_d = \pi$

Lewi's equation,

Tangential force on tooth,

$$F_t = \pi \sigma_b \times b \times m \times y$$

where y - Lewi's form factor

$$y = \frac{t^2}{6h P_d}$$

UNIT-4

Spring Terminology:

1. Solid length,

$$L_s = n d$$

2. Free length,

$$L_f = L_s + \delta_{\max} + 0.15 \delta_{\max}$$

3. Spring Index,

$$C = \frac{D}{d}$$

4. Spring rate / Spring constant / stiffness (k),

$$k = \frac{W}{\delta}$$

5. Pitch,

$$p = \frac{\text{Free length}}{n-1} = \frac{L_f - L_d}{n} + d$$

Stress developed in helical spring:

Torsional shear stress induced in wire,

$$\tau_1 = \frac{8WD}{\pi d^3} \text{ or } \tau_1 = \frac{8WC}{\pi d^2}$$

Direct shear stress developed in helical spring,

$$\tau_2 = \frac{4W}{\pi d^2}$$

Resultant shear stress induced in wire,

$$\tau = \tau_1 \pm \tau_2$$

Maximum shear stress induced in wire,

$$\tau_{\max} = \frac{8WD}{\pi d^3} \left[1 + \frac{d}{2D} \right] = \frac{8WD}{\pi d^3} \left[1 + \frac{1}{2C} \right] = \frac{8WD}{\pi d^3} k_s$$

Where $k_s = 1 + \frac{1}{2C}$ k_s - Shear stress factor

Including curvature stress,

$$\tau_{\max} = k \frac{8WD}{\pi d^3}$$

where k - Wahl's factor

$$k = \frac{4c-1}{4c-4} + \frac{0.615}{c}$$

Deflection in spring;

1. Total active length of wire,

$$l = \pi D n$$

2. Axial deflection of spring,

$$\delta = \theta \times \frac{D}{2}$$

$$\delta = \frac{16WD^3 n}{Gd^4}$$

Deflection per unit active coil,

$$\frac{\delta}{n} = \frac{8WD^3}{Gd^4}$$

3. Spring constant

$$k = \frac{Gd^4}{8D^3 n}$$

$$\text{or } k = \frac{Gd}{8c^3 n}$$

$$c = \frac{D}{d}$$

Energy stored in helical spring of circular wire;

$$U = \frac{\tau^2}{4k^2 G} \times \left(\pi D n \times \frac{\pi d^2}{4} \right)$$

$$U = \frac{\tau^2}{4k^2 G} \times \text{Volume of Spring}$$

$$\left[\begin{array}{l} \text{Area} \times \text{length} = \text{Volume} \\ \frac{\pi d^2}{4} \times \pi D n = \text{Volume} \\ \text{of Spring} \end{array} \right]$$

Series connection of spring,

$$\textcircled{i} W = W_1 = W_2$$

$$\textcircled{ii} \delta = \delta_1 + \delta_2$$

$$\therefore \frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$$

$$k = \frac{k_1 k_2}{k_1 + k_2}$$

Parallel connection of spring,

$$\textcircled{i} W = W_1 + W_2$$

$$\textcircled{ii} \delta = \delta_1 = \delta_2$$

$$\therefore k = k_1 + k_2$$

Terminology of Power Screw

1. Nominal Diameter

$$D_o = D + P$$

2. Core diameter

$$D_c = D - P$$

3. Mean diameter

$$D = \frac{D_o + D_c}{2}$$

4. Pitch, P = Central Distance between two adjacent thread

5. Lead,

$$l = nP$$

n - no. of thread

P - Pitch

6. Helix angle (α),

$$\tan \alpha = \frac{l}{\pi d}$$

Design procedure of screw jack;

Step-1 Calculate the core diameter (d_c) of screw spindle

$$\sigma_c = \frac{W}{\frac{\pi}{4} d_c^2} \quad ; \quad d_c = \sqrt{\frac{4W}{\pi \sigma_c}}$$

Step-2 Calculate outer dia (d_o) and mean dia (d) of screw spindle

Outer dia. , $d_o = d_c + p$

Mean dia. , $d = \frac{d_o + d_c}{2}$

Step-3 Calculate the helix angle,

$$\tan \alpha = \frac{p}{\pi d} \quad ; \quad \alpha = \tan^{-1} \frac{p}{\pi d}$$

Step-4 Calculate the torque

$$T_i = W \tan(\alpha + \theta) \times \frac{d}{2}$$

Step-5 Calculate the compressive stress

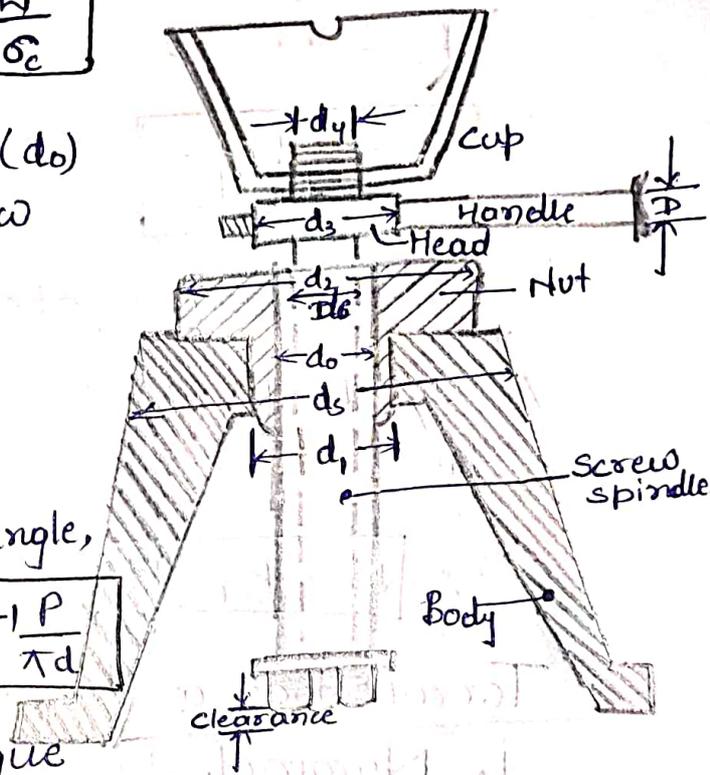
$$\sigma_c = \frac{4W}{\pi d_c^2}$$

Step-6 Calculate the shear stress

$$\tau = \frac{16 T_i}{\pi d_c^3}$$

Calculate the principle stress

$$\sigma_{max} = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$



UNIT-5

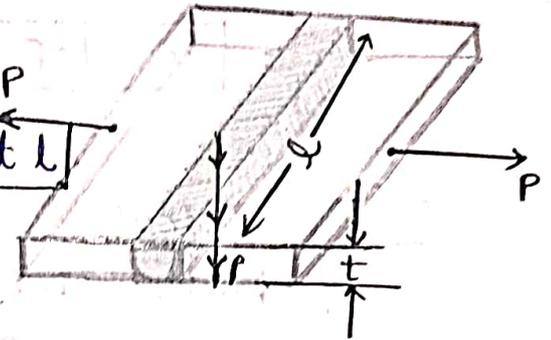
Strength of butt joint ;

When tensile or compressive force/stress is given,

$$\sigma_t \text{ or } \sigma_c = \frac{P}{t \times l}$$

$$P = \sigma_t (t \times l)$$

$$\text{or } P = \sigma_c t l$$



When shear strength is given,

$$\tau_{max} = \frac{P}{t \times l}$$

$$P = \tau_{max} (t \times l)$$

1. Transverse fillet joint ;

If shear stress is given,

for maximum load (single transverse fillet),

$$P = 0.828 h \times l \times \tau_{max}$$

for maximum load in double transverse fillet,

$$P = 2 \times 0.828 h \times l \times \tau_{max}$$

If tensile stress is given,

for maximum strength,

$$P = 2 \times 0.707 \times h \times l \times \sigma_t$$

2. Parallel or longitudinal fillet joint ;

Max^m load acting on single parallel fillet,

$$P = 0.707 h \times l \times \tau_{max}$$

Max^m load acting on single parallel fillet,

$$P = 2 \times 0.707 h \times l \times \tau_{max}$$

$$P_{\text{transverse}} = 1.17 P_{\text{parallel}}$$

Design Procedure of Rivet Joint;

Step-1 Calculate the margin by applying the relation,

$$m = 1.5d$$

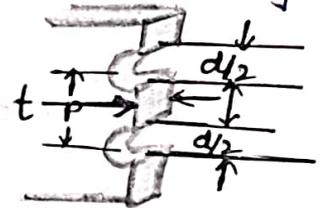
d - dia of rivet

$$d = 6\sqrt{t}$$

Step-2 Calculate the tearing strength (tensile) strength of plate by applying the relation,

$$\sigma_t = \frac{P_t}{(P-d)t}$$

P - Pitch



$$P_t = \sigma_t t (P-d)$$

If $P_t > P(\text{given})$ [Applied Load] then dimension are in safe condition.

Step 3 Calculate the shearing resistance of the rivet,

$$P_s = n \times \frac{\pi}{2} d^2 \tau_{\text{given}} \quad \text{for double shear}$$

$$P_s = n \times 1.875 \times \frac{\pi}{4} d^2 \tau_{\text{given}}$$

Step 4 Calculate the crushing resistance of rivet joint,

$$P_c = n d t \sigma_c$$

Step 5 Calculate efficiency of rivet joint

$$\eta = \frac{\text{least of } P_t \text{ or } P_s \text{ or } P_c}{\text{Strength of solid plate without hole (P)}}$$

$$P_t = \sigma_t t (P-d)$$

$$P_s = n \frac{\pi}{2} d^2 \tau_{\text{given}}$$

$$P_c = n d t \sigma_c$$

$$P = p \times t \times \sigma_c$$

for butt joint,

Step 6 Calculate the thickness of cover plate by the relation,

$$t_1 = 1.125 t$$

for single cover

$$t_1 = 0.625 t$$

for double cover